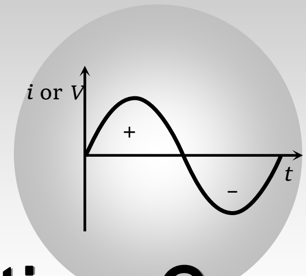
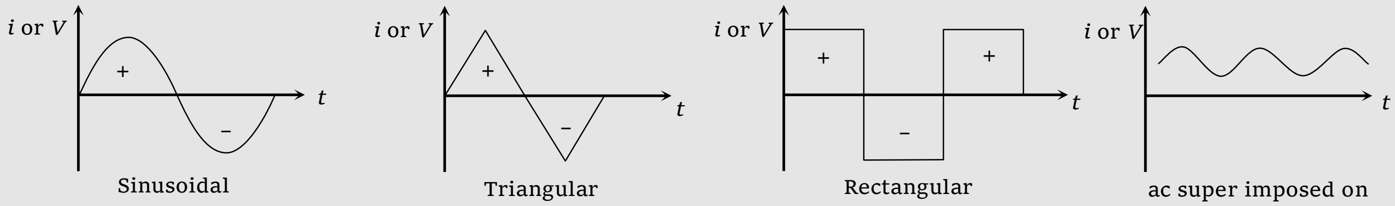


Alternating Current

An alternating quantity (current i or voltage V) is one whose magnitude changes continuously with time between zero and a maximum value and whose direction reverses periodically.

Some graphical representation for alternating quantities



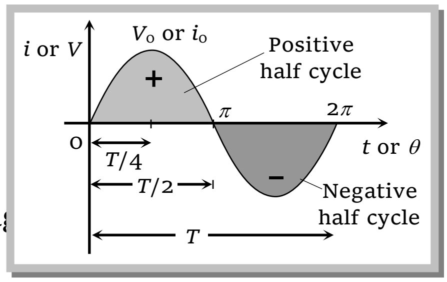
Equation of Alternating Quantities (i or V)

When a coil is rotated rapidly in a strong magnetic field, magnetic flux linked with the coil changes. As a result an emf is induced in the coil and induced current flows through the circuit. These voltage and current are known as alternating voltage and current

(1) **Equation :** Alternating current or voltage varying as sine function can be written as

$$i = i_0 \sin \omega t = i_0 \sin 2\pi\nu t = i_0 \sin \frac{2\pi}{T} t$$

and $V = V_0 \sin \omega t = V_0 \sin 2\pi \nu t = V_0 \sin \frac{2\pi}{T} t$



where i and V = Instantaneous values of current and voltage

i_0 and V_0 = Peak values of current and voltage

ω = Angular frequency in *rad/sec*, ν = Frequency in *Hz* and T = time period

(2) **About cycle**

(i) The time taken to complete one cycle of variations is called the periodic time or time period.

(ii) Alternating quantity is positive for half the cycle and negative for the rest half. Hence average value of alternating quantity (i or V) over a complete cycle is zero.

(iii) Area under the positive half cycle is equal to area under negative cycle.

(iv) The value of alternating quantity is zero or maximum 2ν times every second. The direction also changes 2ν times every second.

(v) Generally sinusoidal waveform is used as alternating current/voltage.

(vi) At $t = \frac{T}{4}$ from the beginning, i or V reaches to their maximum value.

Note : ❑ If instantaneous current i (or voltage V) becomes $1/n$ times of it's peak value in time t then $t = \frac{T}{2\pi}\sin^{-1}\left(\frac{1}{n}\right)$ second.

Important Values of Alternating Quantities

(1) **Peak value (i_0 or V_0)**

The maximum value of alternating quantity (i or V) is defined as peak value or amplitude.

(2) **Mean square value ($\overline{V^2}$ or $\overline{i^2}$)**

The average of square of instantaneous values in one cycle is called mean square value. It is always positive for one complete cycle. *e.g.* $\overline{V^2} = \frac{1}{T}\int_0^T V^2 dt = \frac{V_0^2}{2}$ or $\overline{i^2} = \frac{i_0^2}{2}$

(3) **Root mean square (r.m.s.) value**

Root of mean of square of voltage or current in an *ac* circuit for one complete cycle is called *r.m.s.* value. It is denoted by V_{rms} or i_{rms}

$$i_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + ...}{n}} = \sqrt{\overline{i^2}} = \sqrt{\frac{\int_0^T i^2 dt}{\int_0^T dt}} = \frac{i_0}{\sqrt{2}} = 0.707\, i_0 = 70.7\% \text{ of } i_0$$

similarly $V_{rms} = \frac{V_0}{\sqrt{2}} = 0.707\ V_0 = 70.7\%$ of V_0

(i) The *r.m.s.* value of alternating current is also called virtual value or effective value.

(ii) In general when values of voltage or current for alternating circuits are given, these are *r.m.s.* value.

(iii) ac ammeter and voltmeter are always measure *r.m.s.* value. Values printed on *ac* circuits are *r.m.s.* values.

(iv) In our houses *ac* is supplied at 220 *V*, which is the *r.m.s.* value of voltage. It's peak value is $\sqrt{2} \times 200 = 311\, V$.

(v) *r.m.s.* value of *ac* is equal to that value of *dc*, which when passed through a resistance for a given time will produce the same amount of heat as produced by the alternating current when passed through the same resistance for same time.

Note : ❑ *r.m.s.* value of a complex current wave (*e.g.* $i = a \sin\omega t + b \cos\omega t$) is equal to the square root of the sum of the squares of the *r.m.s.* values of it's individual components *i.e.*

$$i_{rms} = \sqrt{\left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{b}{\sqrt{2}}\right)^2} = \frac{1}{\sqrt{2}}\left(\sqrt{a^2 + b^2}\right).$$

(4) **Mean or Average value (i_{av} or V_{av})**

The average of instantaneous values of current or voltage in one cycle is called it's mean value. The average value of alternating quantity for one complete cycle is zero.

The average value of *ac* over half cycle (t = 0 to $T/2$)

$$i_{av} = \frac{\int_0^{T/2} i\,dt}{\int_0^{T/2} dt} = \frac{2i_0}{\pi} = 0.637\, i_0 = 63.7\% \text{ of } i_0, \text{ Similarly } V_{av} = \frac{2V_0}{\pi} = 0.637\, V_0 = 63.7\% \text{ of } V_0.$$

Specific Examples

Currents	Average value (For complete cycle)	Peak value	*r.m.s.* value	Angular frequency
$i = i_0 \sin\omega t$	0	i_0	$\frac{i_0}{\sqrt{2}}$	ω
$i = i_0 \sin\omega t \cos \omega t$	0	$\frac{i_0}{2}$	$\frac{i_0}{2\sqrt{2}}$	2ω
$i = i_0 \sin\omega t + i_0 \cos\omega t$	0	$\sqrt{2}\, i_0$	i_0	ω

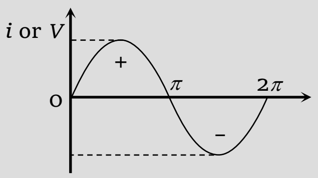
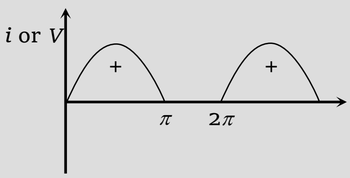
(5) **Peak to peak value**

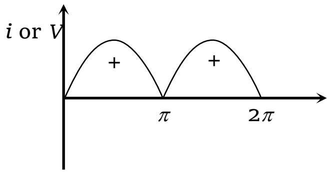
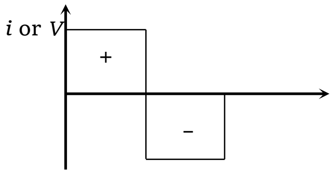
It is equal to the sum of the magnitudes of positive and negative peak values

$\therefore$ Peak to peak value = $V_0 + V_0 = 2V_0 = 2\sqrt{2}\, V_{rms} = 2.828\, V_{rms}$

(6) **Peak factor and form factor**

The ratio of *r.m.s.* value of *ac* to it's average during half cycle is defined as form factor. The ratio of peak value and *r.m.s.* value is called peak factor

Nature of wave form	Wave form	*r.m.s.* value	average value	Form factor $R_f = \frac{\text{r.m.s. value}}{\text{Average value}}$	Peak factor $R_p = \frac{\text{Peak value}}{\text{r.m.s. value}}$
Sinusoidal		$\frac{i_0}{\sqrt{2}}$	$\frac{2}{\pi} i_0$	$\frac{\pi}{2\sqrt{2}} = 1.11$	$\sqrt{2} = 1.41$
Half wave rectified		$\frac{i_0}{2}$	$\frac{i_0}{\pi}$	$\frac{\pi}{2} = 1.57$	2

Full wave rectified		$\frac{i_0}{\sqrt{2}}$	$\frac{2i_0}{\pi}$	$\frac{\pi}{2\sqrt{2}}$	$\sqrt{2}$
Square or Rectangular		i_0	i_0	1	1

Phase

Physical quantity which represents both the instantaneous value and direction of alternating quantity at any instant is called it's phase. It's a dimensionless quantity and it's unit is radian.

If an alternating quantity is expressed as $X = X_0 \sin(\omega t \pm \phi_0)$ then the argument of $\sin(\omega t + \phi)$ is called it's phase. Where ωt = instantaneous phase (changes with time) and ϕ_0 = initial phase (constant *w.r.t.* time)

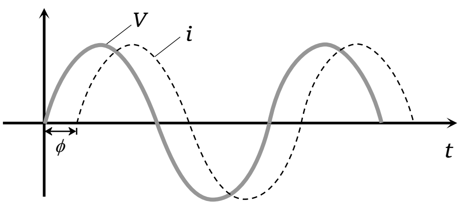
(1) **Phase difference** (Phase constant)

The difference between the phases of currents and voltage is called phase difference. If alternating voltage and current are given by $V = V_0 \sin(\omega t + \phi_1)$ and $i = i_0 \sin(\omega t + \phi_2)$ then phase difference $\phi = \phi_1 - \phi_2$ (relative to current) or $\phi = \phi_2 - \phi_1$ (relative to voltage)

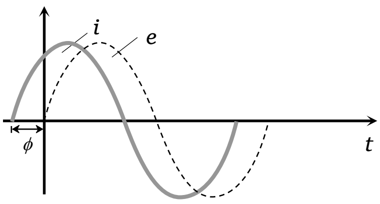
Note : ❑ Phase difference, generally is given relative to current.

❑ The quantity with higher phase is supposed to be leading and the other quantity is taken to be lagging.

(2) **Graphical representation**



Voltage $(V) = V_0 \sin \omega t$
Current $(i) = i_0 \sin(\omega t - \phi)$
Phase difference = 0 – (– ϕ) = + ϕ
i.e. voltage is leading by an angle (+ ϕ) *w.r.t.* current



Voltage $(V) = V_0 \sin \omega t$
Current $(i) = i_0 \sin(\omega t + \phi)$
Phase difference = 0 – (+ ϕ) = – ϕ
i.e. voltage is leading by an angle (– ϕ) *w.r.t.* current

(3) **Time difference**

If phase difference between alternating current and voltage is ϕ then time difference between them is given as

$$\text{T.D.} = \frac{T}{2\pi} \times \phi$$

(4) **Phasor and phasor diagram**

The study of *ac* circuits is much simplified if we treat alternating current and alternating voltage as vectors with the angle between the vectors equals to the phase difference between the current and voltage. The current and voltage are more appropriately called phasors. A diagram representing alternating current and alternating voltage (of same frequency) as vectors (phasors) with the phase angle between them is called a phasor diagram.

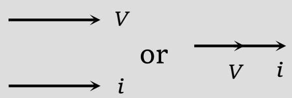
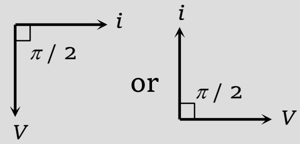
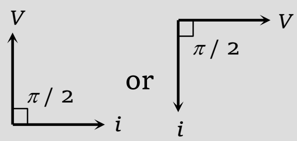
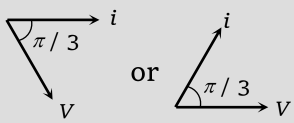
While drawing phasor diagram for a pure element (*e.g.* R, L or C) either of the current or voltage can be plotted along X-axis.

But when phasor diagram for a combination of elements is drawn then quantity which remains constant for the combination must be plotted along X-axis so we observe that

(a) In series circuits current has to be plotted along X-axis.

(b) In parallel circuits voltage has to be plotted along X-axis.

Specific Examples

Equation of V and i	Phase difference ϕ	Time difference T.D.	Phasor diagram
$V = V_0 \sin\omega t$ $i = i_0 \sin\omega t$	0	0	
$V = V_0 \sin\omega t$ $i = i_0 \sin(\omega t + \frac{\pi}{2})$	$-\frac{\pi}{2}$	$\frac{T}{4}$	
$V = V_0 \sin\omega t$ $i = i_0 \sin(\omega t - \frac{\pi}{2})$	$+\frac{\pi}{2}$	$\frac{T}{4}$	
$V = V_0 \sin\omega t$ $i = i_0 \sin(\omega t + \frac{\pi}{3})$	$-\frac{\pi}{3}$	$\frac{T}{6}$	

Measurement of Alternating Quantities

Alternating current shows heating effect only, hence meters used for measuring *ac* are based on heating effect and are called hot wire meters (Hot wire ammeter and hot wire voltmeter)

***ac* measurement**	***dc* measurement**
(1) All ac meters read *r.m.s.* value.	(1) All dc meters read average value
(2) All ac meters are based on heating effect of current.	(2) All dc meters are based on magnetic effect of current
(3) Deflection in hot wire meters : $\theta \propto i_{rms}^2$  (non-linear scale)	(3) Deflection in *dc* meters : $\theta \propto i$  (Linear scale)

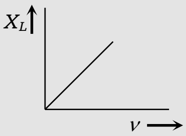
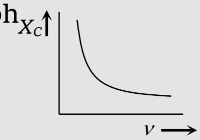
Note : ❑ *ac* meters can be used in measuring *ac* and *dc* both while *dc* meters cannot be used in measuring *ac* because the average value of alternating current and voltage over a full cycle is zero.

Terms Related to ac Circuits

(1) **Resistance (*R*) :** The opposition offered by a conductor to the flow of current through it is defined as the resistance of that conductor. Reciprocal of resistance is known as conductance (*G*) *i.e.* $G = \frac{1}{R}$

(2) **Impedance (*Z*) :** The opposition offered by the capacitor, inductor and conductor to the flow of ac through it is defined as impedance. It's unit is *ohm*(Ω). $Z = \frac{V_0}{i_0} = \frac{V_{rms}}{i_{rms}}$

(3) **Reactance (*X*) :** The opposition offered by inductor or capacitor or both to the flow of ac through it is defined as reactance. It is of following two type –

Inductive reactance (X_L)	**Capacitive reactance (X_C)**
(i) Offered by inductive circuit	(i) Offered by capacitive circuit
(ii) $X_L = \omega L = 2\pi\nu L$	(ii) $X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$
(iii) $\nu_{dc} = 0$ so for dc, $X_L = 0$	(iii) For dc $X_C = \infty$
(iv) X_L-ν Graph 	(iv) X_C - ν Graph 

Note : ❑ Resultant reactance of *LC* circuit is defined as $X = X_L \sim X_C$.

(4) **Admittance (Y) :** Reciprocal of impedance is known as admittance $\left(Y=\frac{1}{Z}\right)$. It's unit is *mho*.

(5) **Susceptance (S) :** the reciprocal of reactance is defined as susceptance $\left(S=\frac{1}{X}\right)$. It is of two type

(i) inductive susceptance $S_L=\frac{1}{X_L}=\frac{1}{2\pi\nu L}$ and (ii) Capacitive susceptance, $S_C=\frac{1}{X_C}=\omega C=2\pi\nu C$.

Power and Power Factor

The power is defined as the rate at which work is being done in the circuit.

In dc circuits power is given by $P = Vi$. But in ac circuits, since there is some phase angle between voltage and current, therefore power is defined as the product of voltage and that component of the current which is in phase with the voltage.

Thus $\boldsymbol{P=Vi\cos\phi}$; where V and i are *r.m.s.* value of voltage and current.

(1) **Types of power**

There are three terms used for power in an ac circuit

(i) **Instantaneous power :** Suppose in a circuit $V=V_0\sin\omega t$ and $i=i_0\sin(\omega t+\phi)$ then $P_{\text{instantaneous}}=Vi=V_0 i_0\sin\omega t\sin(\omega t+\phi)$

(ii) **Average power (True power) :** The average of instantaneous power in an ac circuit over a full cycle is called average power. It's unit is *watt* *i.e.*

$$P_{av}=\overline{P}_{inst}\Rightarrow P_{av}=V_{rms}i_{rms}\cos\phi=\frac{V_0}{\sqrt{2}}.\frac{i_0}{\sqrt{2}}\cos\phi=\frac{1}{2}V_0 i_0\cos\phi=i_{rms}^2R=\frac{V_{rms}^2R}{Z^2}$$

(iii) **Apparent or virtual power :** The product of apparent voltage and apparent current in an electric circuit is called apparent power. This is always positive

$$P_{app}=V_{rms}i_{rms}=\frac{V_0 i_0}{2}$$

(2) **Power factor :** It may be defined as

(i) Cosine of the angle of lag or lead

(ii) The ratio $\frac{R}{Z}=\frac{\text{Resistance}}{\text{Impedance}}$

(iii) The ratio $\frac{\text{True power}}{\text{Apparent power}}=\frac{W}{VA}=\frac{kW}{kVA}=\cos\phi$

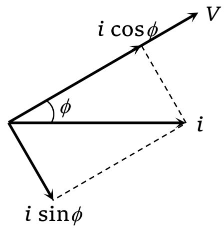
Note :
- ❑ Power factor is a dimensionless quantity and it's value lies between 0 and 1.
- ❑ For a pure resistive circuit $R = Z \Rightarrow$ p.f. = $\cos\phi = 1$

Wattless Current

In an ac circuit $R = 0 \Rightarrow \cos\phi = 0$ so $P_{av} = 0$ *i.e.* in resistance less circuit the power consumed is zero. Such a circuit is called the wattless circuit and the current flowing is called the wattless current.

or

The component of current which does not contribute to the average power dissipation is called wattless current



(i) The average of wattless component over one cycle is zero

(ii) Amplitude of wattless current = $i_0 \sin\phi$

and r.m.s. value of wattless current = $i_{rms} \sin\phi = \frac{i_0}{\sqrt{2}} \sin\phi$.

It is quadrature (90°) with voltage

Note : ❑ The component of ac which remains in phase with the alternating voltage is defined as the effective current. The peak value of effective current is $i_0 \cos\phi$ and it's r.m.s. value is $i_{rms} \cos\phi = \frac{i_0}{\sqrt{2}} \cos\phi$.

Concepts

☞ *If ac is produced by a generator having a large number of poles then it's frequency* $\nu = \frac{\text{Number of poles} \times \text{rotation per second}}{2} = \frac{P \times n}{2}$

Where P is the number of poles; n is the rotational frequency of the coil.

☞ *Alternating current in electric wires, bulbs etc. flows 50 times in one direction and 50 times in the opposite direction in 1 second. Since in one cycle the current becomes zero twice, hence a bulb lights up 100 times and is off 100 times in one second (50 cycles) but due to persistence of vision, it appears lighted continuously.*

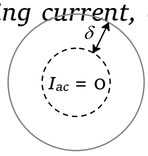
☞ *ac is more dangerous than dc.*

☞ *The rate of change of ac is minimum at that instant when they are near their peak values.*

☞ *ac equipments such as electric motors, are more durable and convenient compared to dc equipments.*

☞ ***Skin Effect***

A direct current flows uniformly throughout the cross-section of the conductor. An alternating current, on the other hand, flows



mainly along the surface of the conductor. This effect is known as skin effect. the reason is that when alternating current flows through a conductor, the flux changes in the inner part of the conductor are higher. Therefore the inductance of the inner part is higher than that of the outer part. Higher the frequency of alternating current, more is the skin effect.

The depth upto which ac current flows through a wire is called skin depth (δ).

☞ *Comparison of electricity in India and America*

India	America
50 *Hz*	60 *Hz*
220 *V*	110 *V*
R	*R* / 4

$R = \frac{V_R^2}{P_R} \Rightarrow R \propto V_R^2$ (V_R = rated voltage, P_R = rated power)

Example

Example: 1 The equation of an alternating current is $i = 50\sqrt{2} \times \sin 400\,\pi t$ *ampere* then the frequency and the root mean square of the current are respectively

(a) 200 *Hz*, 50 *amp* (b) 400 π *Hz*, $50\sqrt{2}$ *amp* (c) 200 *Hz*, $50\sqrt{2}$ *amp* (d) 50 *Hz*, 200 *amp*

Solution: (a) Comparing the given equation with $i = i_0 \sin \omega t$

$\Rightarrow \omega = 400\ \pi \Rightarrow 2\pi\nu = 400\pi \Rightarrow \nu = 200\ Hz$. Also $i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{50\sqrt{2}}{2} = 50\ A$.

***Example*: 2** If the frequency of an alternating current is 50 *Hz* then the time taken for the change from zero to positive peak value and positive peak value to negative peak value of current are respectively

(a) 1/200 *sec*, 1/ 100 *sec* (b) 1/ 100 *sec*, 1/200 *sec* (c) 200 *sec*, 100 *sec*(d)

Solution: (a) Time take to reach from zero to peak value $t = \frac{T}{4} = \frac{1}{4\nu} = \frac{1}{4 \times 50} = \frac{1}{200} sec$

Time take for the change from positive peak to negative peak $t' = \frac{T}{2} = \frac{1}{2\nu} = \frac{1}{2 \times 50} = \frac{1}{100} sec$.

***Example*: 3** What will be the equation of ac of frequency 75 *Hz* if its *r.m.s.* value is 20 *A*

(a) $i = 20 \sin 150\ \pi\ t$ (b) $i = 20\sqrt{2} \sin(150\ \pi\ t)$ (c) $i = \frac{20}{\sqrt{2}} \sin(150\ \pi\ t)$ (d) $i = 20\sqrt{2} \sin(75\ \pi\ t)$

Solution: (b) By using $i = i_0 \sin \omega t = i_0 \sin 2\pi\nu\ t = i_{rms}\sqrt{2} \sin 2\pi\nu\ t \Rightarrow i = 20\sqrt{2} \sin(150\ \pi\ t)$.

***Example*: 4** At what time (From zero) the alternating voltage becomes $\frac{1}{\sqrt{2}}$ times of it's peak value. Where *T* is the periodic time

(a) $\frac{T}{2} sec$ (b) $\frac{T}{4} sec$ (c) $\frac{T}{8} sec$ (d) $\frac{T}{12} sec$

Solution: (c) By using $V = V_0 \sin \omega t \Rightarrow \frac{V_0}{\sqrt{2}} = V_0 \sin \frac{2\pi t}{T} \Rightarrow \frac{1}{\sqrt{2}} = \sin\left(\frac{2\pi}{T}\right)t \Rightarrow \sin\frac{\pi}{4} = \sin\left(\frac{2\pi}{T}\right)t$

$\Rightarrow \frac{\pi}{4} = \frac{2\pi}{T}t \Rightarrow t = \frac{T}{8} sec.$

***Example*: 5** The peak value of an alternating *e.m.f.* *E* is given by $E = E_0 \cos\omega t$ is 10 *volts* and its frequency is $50\ Hz$. At time $t = \frac{1}{600} sec$, the instantaneous *e.m.f.* is

(a) 10 *V* (b) $5\sqrt{3}V$ (c) 5 *V* (d) 1*V*

Solution: (b) By using $E = E_0 \sin \omega t = 10 \cos 2\pi\nu t = 10 \cos 2\pi \times 50 \times \frac{1}{600} \Rightarrow E = 10 \cos \frac{\pi}{6} = 5\sqrt{3}\ V$

***Example*: 6** The instantaneous value of current in an ac circuit is $i = 2 \sin(100\ \pi\ t + \pi/3)A$. The current at the beginning $(t = 0)$ will be

(a) $2\sqrt{3}A$ (b) $\sqrt{3}A$ (c) $\frac{\sqrt{3}}{2}A$ (d) Zero

Solution: (b) At $t = 0$, $i = 2\sin\left(0 + \frac{\pi}{3}\right) = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}A$.

***Example*: 7** The voltage of an ac source varies with time according to the equation $V = 100 \sin(100\pi t) \cos(100\pi t)$ where *t* is in seconds and *V* is in *volts*. Then

(a) The peak voltage of the source is 100 *volts* (b) The peak voltage of the source is 50 *volts*

(c) The peak voltage of the source is $100/\sqrt{2}\,volts$ (d) The frequency of the source is 50 *Hz*

Solution: (b) The given equation can be written as follows

$V = 50 \times 2\sin 100\pi t\cos 100\pi t = 50\sin 2(100\pi t) = 50\sin 200\pi t$ ($\because$ sin 2θ = 2 sinθ cosθ)

Hence peak voltage $V_0 = 50\,volt$ and frequency $\nu = \frac{200\pi}{2\pi} = 100\,Hz$.

***Example*: 8** If the frequency of ac is 60 *Hz* the time difference corresponding to a phase difference of 60° is

(a) 60 sec (b) 1*sec* (c) $\frac{1}{60}\,sec$ (d) $\frac{1}{360}\,sec$

Solution: (d) Time difference T.D. $= \frac{T}{2\pi}\times\phi \Rightarrow$ T.D. $= \frac{T}{2\pi}\times\frac{\pi}{3} = \frac{T}{6} = \frac{1}{6\nu} = \frac{1}{6\times 60} = \frac{1}{360}\,sec$

***Example*: 9** In an ac circuit, *V* and *i* are given by $V = 100\sin(100t)\,volts$, and $i = 100\sin\left(100t + \frac{\pi}{3}\right)mA$. The power dissipated in circuit is

(a) $10^4\,watt$ (b) 10 *watt* (c) 2.5 *watt* (d) 5 *watt*

Solution: (c) $P = \frac{1}{2}V_0 i_0\cos\phi = \frac{1}{2}\times 100\times(100\times 10^{-3})\times\cos\left(\frac{\pi}{3}\right) = 2.5\,watt.$

***Example*: 10** In a circuit an alternating current and a direct current are supplied together. The expression of the instantaneous current is given as $i = 3 + 6\sin\omega t$. Then the *r.m.s.* value of the current is

(a) 3*A* (b) 6*A* (c) $3\sqrt{2}\,A$ (d) $3\sqrt{3}\,A$

Solution: (d) The given current is a mixture of a dc component of 3*A* and an alternating current of maximum value 6*A*

Hence *r.m.s.* value $= \sqrt{(dc)^2 + (r.m.s.\text{ value of ac})^2} = \sqrt{(3)^2 + \left(\frac{6}{\sqrt{2}}\right)^2} = \sqrt{(3)^2 + (3\sqrt{2})^2} = 3\sqrt{3}A$.

***Example*: 11** The *r.m.s.* value of the alternating *e.m.f.* $E = (8\sin\omega t + 6\sin 2\omega t)\,V$ is

(a) 7.05 *V* (b) 14.14 *V* (c) 10 *V* (d) 20 *V*

Solution: (a) Peak value $V_0 = \sqrt{(8)^2 + (6)^2} = 10\,volt$ so $v_{rms} = \frac{10}{\sqrt{2}} = 5\sqrt{2} = 7.05\,volt$.

***Example*: 12** Voltage and current in an ac circuit are given by $V = 5\sin\left(100\pi t - \frac{\pi}{6}\right)$ and $i = 4\sin\left(100\pi t + \frac{\pi}{6}\right)$

[Kerala (Engg.) 2001]

(a) Voltage leads the current by 30° (b) Current leads the voltage by 30°

(c) Current leads the voltage by 60° (d) Voltage leads the current by 60°

Solution: (c) Phase difference relative to current $\Delta\phi = -\frac{\pi}{6} - \frac{\pi}{6} = -\frac{\pi}{3}$

In degree $\Delta\phi = -60^o$ *i.e.* voltage lag behind the current by 60° or current leads the voltage by 60°.

***Example*: 13** The instantaneous values of current and potential difference in an alternating circuit are $i = \sin\omega t$ and $E = 100\cos\omega t$ respectively. *r.m.s.* value of wattless current (in *amp*) in the circuit is

(a) 1 (b) $1/\sqrt{2}$ (c) 100 (d) Zero

Solution: (b) *r.m.s.* value of wattless current $= \frac{i_0}{\sqrt{2}} \sin\phi$

In this question i_0 = 1 *A* and $\phi = \frac{\pi}{2}$. So *r.m.s.* value of wattless current $= \frac{1}{\sqrt{2}} A$

***Example*: 14** The *r.m.s.* current in an ac circuit is 2 *A*. If the wattless current be $\sqrt{3} A$, what is the power factor

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

Solution: (c) $i_{WL} = i_{rms} \sin\phi \Rightarrow \sqrt{3} = 2\sin\phi \Rightarrow \sin\phi = \frac{\sqrt{3}}{2} \Rightarrow \phi = 60^o$ so p.f. $= \cos\phi = \cos 60^o = \frac{1}{2}$.

***Example*: 15** *r.m.s.* value of alternating current in a circuit is 4 *A* and power factor is 0.5. If the power dissipated in the circuit is 100*W*, then the peak value of voltage in the circuit is

(a) 50 *volt* (b) 70 *volt* (c) 35 *volt* (d) 100 *volt*

Solution: (b) $P = V_{rms}\, i_{rms} \cos\phi \Rightarrow 100 = V_{rms} \times 4 \times 0.5 \Rightarrow V_{rms} = 50\,V$ so $V_0 = \sqrt{2} \times 50 = 70$ *volt*

***Example*: 16** The impedance of an ac circuit is 200 Ω and the phase angle between current and *e.m.f* is 60^o. What is the resistance of the circuit

(a) 50 Ω (b) 100 Ω (c) $100\sqrt{3}\Omega$ (d) 300 Ω

Solution: (b) By using $\cos\phi = \frac{R}{Z} \Rightarrow \cos 60^o = \frac{R}{200} \Rightarrow \frac{1}{2} = \frac{R}{200} \Rightarrow R = 100\ \Omega.$

Tricky example: 1

An ac voltage source of *E* = 150 sin 100 *t* is used to run a device which offers a resistance of 20Ω and restricts the flow of current in one direction only. The *r.m.s.* value of current in the circuit will be

(a) 1.58 *A* (b) 0.98 *A* (c) 3.75 *A* (d) 2.38 *A*

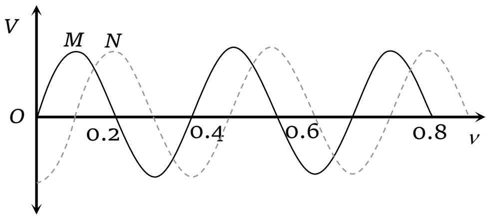
Solution : (c) As current flows in a single direction, the device allows current only during positive half cycle only

$\therefore\ i_{rms} = \frac{i_0}{2} = \frac{V_0}{2R} = \frac{150}{2 \times 20} = 3.75\,A$.

Tricky example: 2

Two sinusoidal voltages of the same frequency are shown in the diagram. What is the frequency, and the phase relationship between the voltages

Frequency in *Hz*	Phase lead of *N* over *M* in radians
(a) 0.4	$-\pi/4$
(b) 2.5	$-\pi/2$
(c) 2.5	$+\pi/2$
(d) 2.5	$-\pi/4$

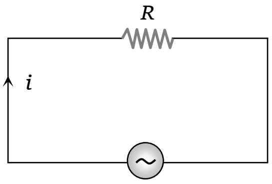
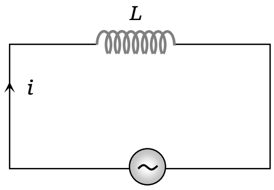
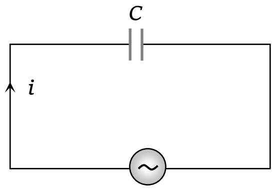
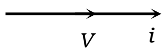
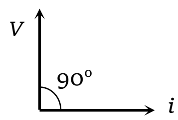
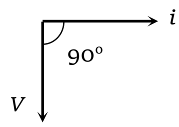


Solution : (b) From the graph shown below. It is clear that phase lead of N over M is $-\frac{\pi}{2}$. Since time period (*i.e.* taken to complete one cycle) = 0.4 *sec*.

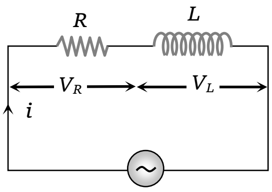
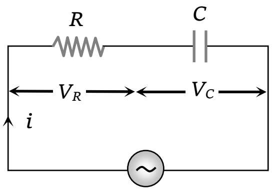
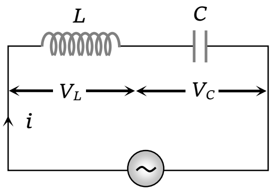
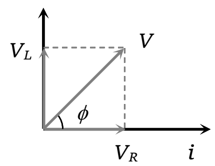
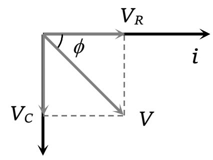
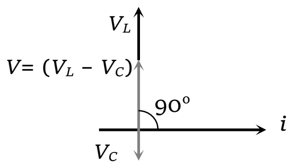
Hence frequency $\nu = \frac{1}{T} = 2.5\ Hz$

Different ac Circuit

(1) ***R*, *L* and *C* circuits**

Circuit characteristics	Purely resistive circuit (*R*-circuit)	Purely inductive circuit (*L*-circuit)	Purely capacitive circuit (*C*-circuit)
(i) Circuit	 $V = V_0 \sin \omega t$	 $V = V_0 \sin \omega t$	 $V = V_0 \sin \omega t$
(ii) Current	$i = i_0 \sin \omega t$	$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right)$	$i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$
(iii) Peak current	$i_0 = \frac{V_0}{R}$	$i_0 = \frac{V_0}{X_L} = \frac{V_0}{\omega_L} = \frac{V_0}{2\pi\nu L}$	$i_0 = \frac{V_0}{X_C} = V_0 \omega C = V_0(2\pi\nu C)$
(iv) Phase difference	$\phi = 0°$	$\phi = 90°$ (or $+\frac{\pi}{2}$)	$\phi = 90°$ (or $-\frac{\pi}{2}$)
(v) Power factor	$\cos\phi = 1$	$\cos\phi = 0$	$\cos\phi = 0$
(vi) Power	$P = V_{rms} i_{rms} = \frac{V_0 i_0}{2}$	$P = 0$	$P = 0$
(vii) Time difference	TD = 0	TD = $\frac{T}{4}$	TD = $\frac{T}{4}$
(viii) Leading quantity	Both are in same phase	Voltage	Current
(ix) Phasor diagram			

(2) ***RL*, *RC* and *LC* circuits**

Circuit characterstics	***RL*-circuit**	***RC*-circuit**	***LC*-circuit**
(i) Circuit	 $V_R = iR$, $V_L = iX_L$ $V = V_0 \sin \omega t$	 $V_R = iR$, $V_C = iX_C$ $V = V_0 \sin \omega t$	 $V_L = iX_L$, $V_C = iX_C$ $V = V_0 \sin \omega t$
(ii) Current	$i = i_0 \sin(\omega t - \phi)$	$i = i_0 \sin(\omega t + \phi)$	$i = i_0 \sin\left(\omega t \pm \frac{\pi}{2}\right)$
(iii) Peak current	$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + X_L^2}}$ $= \frac{V_0}{\sqrt{R^2 + 4\pi^2\nu^2L^2}}$	$i_0 = \frac{V_0}{Z} = \frac{V_0}{\sqrt{R^2 + X_C^2}}$ $= \frac{V_0}{\sqrt{R^2 + \frac{1}{4\pi^2\nu^2C^2}}}$	$i_0 = \frac{V_0}{Z} = \frac{V_0}{X_L - X_C}$ $= \frac{V_0}{\omega L - \frac{1}{\omega C}}$
(iv) Phasor diagram			
(v) Applied voltage	$V = \sqrt{V_R^2 + V_L^2}$	$V = \sqrt{V_R^2 + V_C^2}$	$V = V_L - V_C$
(vi) Impedance	$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2L^2}$ $= \sqrt{R^2 + 4\pi^2\nu^2L^2}$	$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$	$Z = X_L - X_C = X$
(vii) Phase difference	$\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{\omega L}{R}$	$\phi = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\frac{1}{\omega CR}$	$\phi = 90°$
(viii) Power factor	$\cos\phi = \frac{R}{\sqrt{R^2 + X_L^2}}$	$\cos\phi = \frac{R}{\sqrt{R^2 + X_C^2}}$	$\cos\phi = 0$
(ix) Leading quantity	Voltage	Current	Either voltage or current

Note : ❑ In *LC* circuit if $X_L = X_C \Rightarrow V_L = V_C$ then resonance occurs and resonant frequency (natural frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ *rad/sec* or $\nu_0 = \frac{1}{2\pi\sqrt{LC}}$ *Hz*.

Example

***Example*: 17** In a resistive circuit R = 10 Ω and applied alternating voltage V = 100 sin 100 πt. Find the following

(i) Peak current (ii) *r.m.s.* current (iii) Average current (iv) (v) Time period (vi) Power factor (vii) (viii) Time difference

Solution: (i) Peak current $i_0 = \frac{V_0}{R} = \frac{100}{10} = 10\ A$

(ii) *r.m.s.* current $i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2} A$

(iii) Average current $i_{av} = \frac{2}{\pi}.i_0 = \frac{2}{\pi}\times 10 = 6.37\ A$

(iv) Frequency $\nu = \frac{\omega}{2\pi} = \frac{100\ \pi}{2\pi} = 50\ Hz$

(v) Time period $T = \frac{1}{\nu} = \frac{1}{50} = 0.02\ sec$

(vi) Phase difference in resistive circuit $\phi = 0$ so p.f. = cosϕ = 1

(vii) Power dissipated in the circuit $P = \frac{1}{2}V_0 i_0 \cos\phi = \frac{1}{2}\times 100\ \times 10\ \times 1 = 500\ W$

(viii) Time difference T.D. $= \frac{T}{2\pi}\times\phi = \frac{T}{2\pi}\times 0 = 0$

***Example*: 18** In a purely inductive circuit if $L = \frac{100}{\pi}\times 10^{-3} H$ and applied alternating voltage is given by V = 100 sin 100 πt. Find the followings

(i) Inductive reactance (ii) Peak value, *r.m.s.* value and average value of current

(iii) Frequency and time period (iv) Power factor and power

(v) Time difference between voltage and current

Solution: (i) $X_L = \omega L = 100\ \pi \times \frac{100}{\pi}\times 10^{-3} = 10\ \Omega$

(ii) $i_0 = \frac{V_0}{X_L} = \frac{100}{10} = 10\ A$; $i_{rms} = \frac{10}{\sqrt{2}} = 5\sqrt{2} A$ and $i_{av} = \frac{2}{\pi}\times 10$ = 6.37 A

(iii) Frequency $\nu = \frac{100\ \pi}{2\pi} = 50\ Hz$ and $T = \frac{1}{50} = 0.02\ sec$

(iv) In purely L-circuit ϕ = 90° so p.f. cosϕ = 0

(v) Time difference T.D. $= \frac{T}{2\pi}\times\frac{\pi}{2} = \frac{T}{4}$.

***Example*: 19** An alternating voltage $E = 200\sqrt{2}\sin(100\ t)$ is connected to a 1 *microfaracd* capacitor through an ac ammeter. The reading of the ammeter shall be

(a) 10 *mA* (b) 20 *mA* (c) 40 *mA* (d) 80 *mA*

Solution: (b) Ammeter reads *r.m.s.* value so $i_{rms} = \frac{V_{rms}}{X_C} = V_{rms}\times\omega\times C$

$$\Rightarrow i_{rms} = \left(\frac{200\sqrt{2}}{\sqrt{2}}\right) \times 100 \times (1 \times 10^{-6}) = 2 \times 10^{-2} = 20\ mA.$$

***Example*: 20** An 120 *volt* ac source is connected across a pure inductor of inductance 0.70 *henry*. If the frequency of the source is 60 *Hz*, the current passing through the inductor is

(a) 4.55 *amps* (b) 0.355 *amps* (c) 0.455 *amps* (d) 3.55 *amps*

Solution: (c) $i_{rms} = \frac{V_{rms}}{X_L} = \frac{V_{rms}}{2\pi\nu L} = \frac{120}{2\pi \times 60 \times 0.7} = 0.455\ A.$

***Example*: 21** The frequency for which a $5\mu F$ capacitor has a reactance of $\frac{1}{1000}$ *ohm* is given by

(a) $\frac{100}{\pi}\ MHz$ (b) $\frac{1000}{\pi}\ Hz$ (c) $\frac{1}{1000}\ Hz$ (d) 1000 *Hz*

Solution: (a) $X_C = \frac{1}{2\pi\nu C} \Rightarrow \nu = \frac{1}{2\pi X_C (C)} = \frac{1}{2\pi \times \frac{1}{1000} \times 5 \times 10^{-6}} = \frac{100}{\pi} MHz.$

***Example*: 22** Let frequency ν = 50 *Hz*, and capacitance *C* = 100μF in an ac circuit containing a capacitor only. If the peak value of the current in the circuit is 1.57 *A*. The expression for the instantaneous voltage across the capacitor will be

(a) $E = 50 \sin(100\pi t - \frac{\pi}{2})$ (b) $E = 100 \sin(50\pi t)$ (c) $E = 50 \sin(100\pi t)$(d)

Solution: (a) Peak value of voltage $V_0 = i_0 X_C = \frac{i_0}{2\pi\nu C} \Rightarrow \frac{1.57}{2 \times 3.14 \times 50 \times 100 \times 10^{-6}} = 50\ V$

Hence if equation of current $i = i_0 \sin\omega t$ then in capacitive circuit voltage is $V = V_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

$$\Rightarrow V = 50\left(\sin 2\pi \times 50\, t - \frac{\pi}{2}\right) = 50 \sin\left(100\,\pi\, t - \frac{\pi}{2}\right)$$

***Example*: 23** In an *LR*-circuit, the inductive reactance is equal to the resistance *R* of the circuit. An *e.m.f.* $E = E_0 \cos(\omega t)$ is applied to the circuit. The power consumed in the circuit is

(a) $\frac{E_0^2}{R}$ (b) $\frac{E_0^2}{2R}$ (c) $\frac{E_0^2}{4R}$ (d) $\frac{E_0^2}{8R}$

Solution: (c) Power consumed $P = E_{rms}\, i_{rms} \cos\phi = E_{rms}\left(\frac{E_{rms}}{Z}\right)\frac{R}{Z} \Rightarrow P = \frac{E_{rms}^2 R}{Z^2}$; where $Z = \sqrt{R^2 + X_L^2}$

Given $X_L = R \Rightarrow Z = \sqrt{2}R$ also $E_{rms} = \frac{E_0}{\sqrt{2}} \Rightarrow P = \frac{E_0^2}{4R}.$

***Example*: 24** A coil of resistance 300 *ohm* and self inductance 1.5 *henry* is connected to an ac source of frequency $\frac{100}{\pi} Hz$. The phase difference between voltage and current is

(a) 0^o (b) 30^o (c) 45^o (d) 60^o

Solution: (c) By using $\tan\phi = \frac{X_L}{R} = \frac{2\pi\nu L}{R} \Rightarrow \tan\phi = \frac{2\pi \times \frac{100}{\pi} \times 1.5}{300} = 1 \Rightarrow \phi = 45°.$

***Example*: 25** The current and voltage in an ac circuit are respectively given by $i = \sin 314\, t$ and $e = 200\,\sin(314\, t + \pi/3)$. If the resistance is 100Ω, then the reactance of the circuit is

(a) $100/\sqrt{3}\Omega$ (b) $100\sqrt{3}\Omega$ (c) 200 Ω (d) $200\sqrt{3}\Omega$

Solution: (b) From the given equation $i_0 = 1A$ and $V_0 = 200\, volt$. Hence $Z = \frac{200}{1} = 200\ \Omega$ also $Z^2 = R^2 + X_L^2$

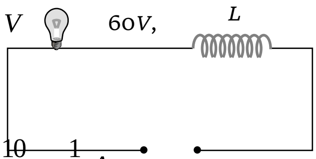
$\Rightarrow (200)^2 = (100)^2 + X_L^2 \Rightarrow X_L = 100\sqrt{3}\,\Omega$.

***Example*: 26** A bulb of 60 *volt* and 10 *watt* is connected with 100 *volt* of ac source with an inductance coil in series. If bulb illuminates with it's full intensity then value of inductance of coil is (ν= 60 *Hz*) **[RPMT 1995]**

(a) 1.28 *H* (b) 2.15 *H* (c) 3.27 *H* (d) 3.89 *H*

Solution: (a) Resistance of the bulb $R = \frac{60 \times 60}{10} = 360\ \Omega$.

For maximum illumination, voltage across the bulb $V_{Bulb} = V_R = 60\, V$



By using $V = \sqrt{V_R^2 + V_L^2} \Rightarrow (100)^2 = (60)^2 + V_L^2 \Rightarrow V_L = 80\, V$

Current through the inductance (L) = Current through the bulb $= \frac{10}{60} = \frac{1}{6}A$

Also $V_L = iX_L = i(2\pi\nu L) \Rightarrow L = \frac{V_L}{(2\pi\nu)i} = \frac{80}{2 \times 3.14 \times 60 \times \frac{1}{6}} = 1.28\, H$.

***Example*: 27** When 100 *volt* dc is applied across a solenoid, a current of 1.0 *amp* flows in it. When 100 *volt* ac is applied across the same coil, the current drops to 0.5 *amp*. If the frequency of ac source is 50 *Hz* the impedance and inductance of the solenoid are

(a) 200 ohms and 0.5 henry (b) 100 ohms and 0.86 henry

(c) 200 ohms and 1.0 henry (d) 100 ohms and 0.93 henry

Solution: (a) When dc is applied $i = \frac{V}{R} \Rightarrow 1 = \frac{100}{R} \Rightarrow R$ = 100Ω. When ac is applied $i = \frac{V}{Z} \Rightarrow 0.5 = \frac{100}{Z} \Rightarrow Z$ = 200Ω.

Hence $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + 4\pi^2\nu^2L^2} \Rightarrow (200)^2 = (100)^2 + 4\pi^2(50)^2L^2 \Rightarrow L$ = 0.55*H*.

***Example*: 28** In an ac circuit, containing an inductance and a capacitor in series, the current is found to be maximum when the value of inductance is 0.5 henry and a capacitance of $8\,\mu F$. The angular frequency of the input ac voltage must be equal to

(a) 500 *rad/sec* (b) 5×10^4 *rad/sec* (c) 4000 *rad/sec* (d) 5000 *rad/sec*

Solution: (a) Current is maximum *i.e.* the given circuit is in resonance, and at resonance $\omega_0 = \frac{1}{\sqrt{LC}}$

$\Rightarrow \omega_0 = \frac{1}{\sqrt{0.5 \times 8 \times 10^{-6}}} = \frac{1}{2 \times 10^{-3}} = 500\ rad/sec$.

***Example*: 29** A resistance of 40 *ohm* and an inductance of 95.5 *millihenry* are connected in series in a 50 *cycles/second* ac circuit. The impedance of this combination is very nearly

(a) 30 *ohm* (b) 40*ohm* (c) 50 *ohm* (d) 60 *ohm*

Solution: (c) $X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 95.5 \times 10^{-3} = 29.98\,\Omega \approx 30\Omega$

Impedance $Z = \sqrt{R^2 + X_L^2} = \sqrt{(40)^2 + (30)^2} = 50\ \Omega$

***Example*: 30** $\frac{2.5}{\pi}\mu F$ capacitor and 3000-*ohm* resistance are joined in series to an ac source of 200 *volt* and $50 sec^{-1}$ frequency. The power factor of the circuit and the power dissipated in it will respectively

(a) 0.6, 0.06 *W* (b) 0.06, 0.6 *W* (c) 0.6, 4.8 *W* (d) 4.8, 0.6 *W*

Solution: (c) $Z = \sqrt{R^2 + \left(\frac{1}{2\pi\nu C}\right)^2} = \sqrt{(1000)^2 + \frac{1}{\left(2\pi \times 50 \times \frac{2.5}{\pi} \times 10^{-6}\right)^2}} \Rightarrow Z = \sqrt{(3000)^2 + (4000)^2} = 8 \times 10^3\,\Omega$

So power factor $\cos\phi = \frac{R}{Z} = \frac{3000}{5 \times 10^3} = 0.6$ and power $P = V_{rms} i_{rms} \cos\phi = \frac{V_{rms}^2 \cos\phi}{Z} \Rightarrow$

$P = \frac{(200)^2 \times 0.6}{5 \times 10^3} = 4.8W$

***Example*: 31** A telephone wire of length 200 *km* has a capacitance of 0.014 μF per *km*. If it carries an ac of frequency 5 *kHz*, what should be the value of an inductor required to be connected in series so that the impedance of the circuit is minimum

(a) 0.35 *mH* (b) 35 *mH* (c) 3.5 *mH* (d) Zero

Solution: (a) Capacitance of wire $C = 0.014 \times 10^{-6} \times 200 = 2.8 \times 10^{-6} F = 2.8\mu F$

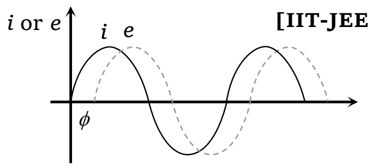
For impedance of the circuit to be minimum $X_L = X_C \Rightarrow 2\pi\nu L = \frac{1}{2\pi\nu C}$

$\Rightarrow L = \frac{1}{4\pi^2\nu^2 C} = \frac{1}{4(3.14)^2 \times (5 \times 10^3)^2 \times 2.8 \times 10^{-6}} = 0.35 \times 10^{-3} H = 0.35\ mH$

Tricky example: 3

When an ac source of e.m.f. $e = E_0 \sin(100\,t)$ is connected across a circuit, the phase difference between the e.m.f. *e* and the current *i* in the circuit is observed to be $\pi/4$, as shown in the diagram. If the circuit consists possibly only of *RC* or *LC* in series, find the relationship between the two elements

[IIT-JEE (Screening) 2003]



(a) $R = 1k\Omega, C = 10\,\mu F$

(b) $R = 1k\Omega, C = 1\mu F$

(c) $R = 1k\Omega, L = 10H$

(d) $R = 1k\Omega, L = 1H$

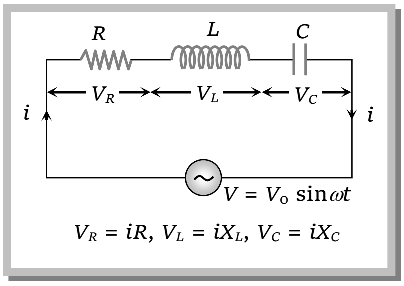
Solution : (a) As the current *i* leads the voltage by $\frac{\pi}{4}$, it is an *RC* circuit, hence $\tan\phi = \frac{X_C}{R} \Rightarrow$

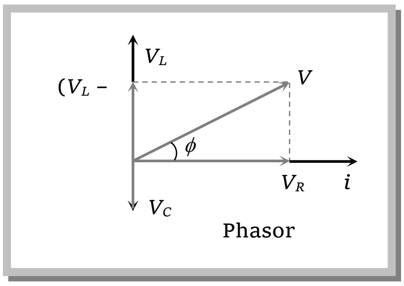
$\tan\frac{\pi}{4} = \frac{1}{\omega CR}$

$\Rightarrow \omega CR = 1$ as $\omega = 100\ rad/sec \Rightarrow CR = \frac{1}{100} sec^{-1}$.

From all the given options only option (a) is correct.

Series *RLC* Circuit





(1) **Equation of current :** $i = i_0 \sin(\omega t \pm \phi)$; where $i_0 = \frac{V_0}{Z}$

(2) **Equation of voltage :** From phasor diagram $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

(3) **Impedance of the circuit :** $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

(4) **Phase difference :** From phasor diagram

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{2\pi\nu L - \frac{1}{2\pi\nu C}}{R}$$

(5) **If net reactance is inductive :** Circuit behaves as *LR* circuit

(6) **If net reactance is capacitive :** Circuit behave as *CR* circuit

(7) **If net reactance is zero :** Means $X = X_L - X_C = 0 \Rightarrow X_L = X_C$. This is the condition of resonance

(8) **At resonance** (series resonant circuit)

(i) $X_L = X_C \Rightarrow Z_{min} = R$ *i.e.* circuit behaves as resistive circuit

(ii) $V_L = V_C \Rightarrow V = V_R$ *i.e.* whole applied voltage appeared across the resistance

(iii) Phase difference : $\phi = 0^\circ \Rightarrow$ p.f. $= \cos\phi = 1$

(iv) Power consumption $P = V_{rms}\, i_{rms} = \frac{1}{2} V_0 i_0$

(v) Current in the circuit is maximum and it is $i_0 = \frac{V_0}{R}$

(vi) These circuit are used for voltage amplification and as selector circuits in wireless telegraphy.

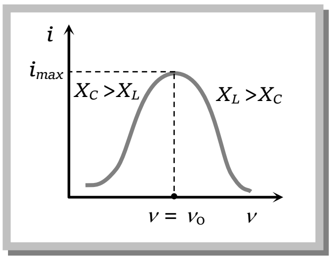
(9) **Resonant frequency** (Natural frequency)

At resonance $X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \frac{rad}{sec} \Rightarrow \nu_0 = \frac{1}{2\pi\sqrt{LC}} Hz$ (or cps)

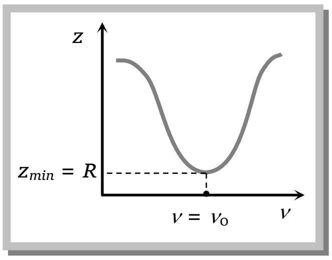
(Resonant frequency doesn't depend upon the resistance of the circuit)

(10) **Different graphs**

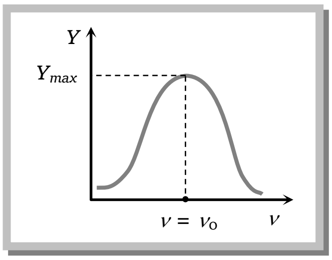
(i) i - ν graph



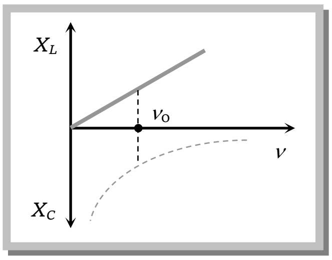
(ii) z - ν graph



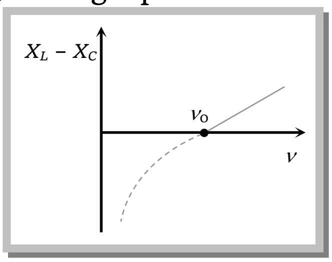
(iii) Y - ν graph



(iv) (X_L, X_C) - ν graph

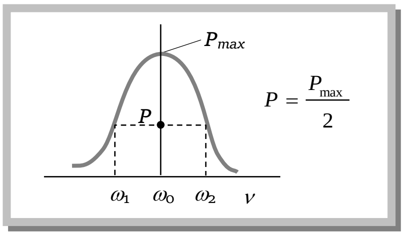


(v) X - ν graph



(11) **Half power frequencies and band width :** The frequencies at which the power in the circuit is half of the maximum power (The power at resonance), are called half power frequencies.

(i) The current in the circuit at half power frequencies (HPF) is $\frac{1}{\sqrt{2}}$ or 0.707 or 70.7% of maximum current (current at resonance).



(ii) There are two half power frequencies.

(a) $\omega_1 \rightarrow$ called lower half power frequency. At this frequency the circuit is capacitive.

(b) $\omega_2 \rightarrow$ called upper half power frequency. It is greater than ω_0. At this frequency the circuit is inductive.

(iii) Band width ($\Delta\omega$) : The difference of half power frequencies ω_1 and ω_2 is called band width ($\Delta\omega$) and $\Delta\omega = \omega_2 - \omega_1$. For series resonant circuit it can be proved $\Delta\omega = \left(\frac{R}{L}\right)$

(12) **Quality factor (Q - factor) of series resonant circuit**

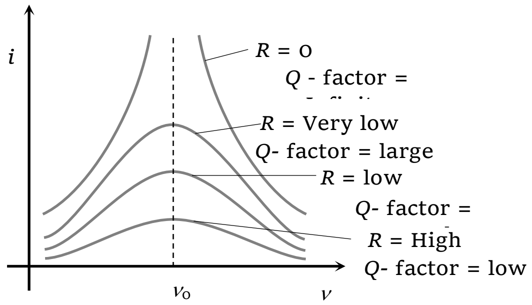
The characteristic of a series resonant circuit is determined by the quality factor (Q - factor) of the circuit.

It defines sharpness of i - ν curve at resonance when Q - factor is large, the sharpness of resonance curve is more and vice-versa.

Q - factor also defined as follows

$$Q\text{ - factor} = 2\pi \times \frac{\text{Maximum energy stored}}{\text{Energy dissipation}} = \frac{2\pi}{T} \times \frac{\text{Maximum energy stored}}{\text{Mean power dissipated}} = \frac{\text{Resonant frequency}}{\text{Band width}} = \frac{\omega_0}{\Delta\omega}$$

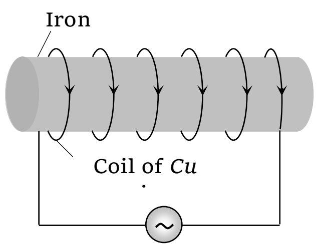
$$Q\text{ - factor} = \frac{V_L}{V_R} \text{ or } \frac{V_C}{V_R} = \frac{\omega_0 L}{R} \text{ or } \frac{1}{\omega_0 CR} \Rightarrow Q\text{ - factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$



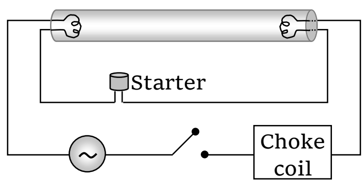
Resonance

Choke Coil

Choke coil (or ballast) is a device having high inductance and negligible resistance. It is used to control current in ac circuits and is used in fluorescent tubes. The power loss in a circuit containing choke coil is least.



Choke coil



Application of choke

(1) It consist of a Cu coil wound over a soft iron laminated core.

(2) Thick Cu wire is used to reduce the resistance (R) of the circuit.

(3) Soft iron is used to improve inductance (L) of the circuit.

(4) The inductive reactance or effective opposition of the choke coil is given by $X_L = \omega L = 2\pi\nu L$

(5) For an ideal choke coil $r = 0$, no electric energy is wasted *i.e.* average power $P = 0$.

(6) In actual practice choke coil is equivalent to a $R - L$ circuit.

(7) Choke coil for different frequencies are made by using different substances in their core.

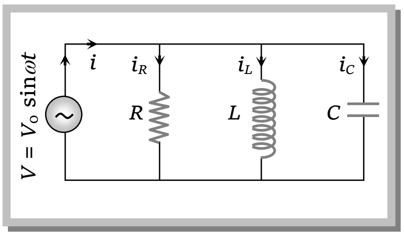
For low frequency L should be large thus iron core choke coil is used. For high frequency ac circuit, L should be small, so air cored choke coil is used.

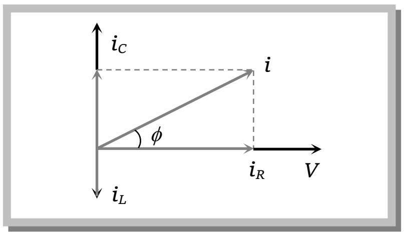
Parallel *RLC* Circuits

$$i_R = \frac{V_0}{R} = V_0 G$$

$$i_L = \frac{V_0}{X_L} = V_0 S_L$$

$$i_C = \frac{V_0}{X_C} = V_0 S_C$$





(1) **Current and phase difference**

From phasor diagram current $i = \sqrt{i_R^2 + (i_C - i_L)^2}$ and phase difference $\phi = \tan^{-1}\frac{(i_C - i_L)}{i_R} = \tan^{-1}\frac{(S_C - S_L)}{G}$

(2) **Admittance (*Y*) of the circuit**

From equation of current $\frac{V_0}{Z} = \sqrt{\left(\frac{V_0}{R}\right)^2 + \left(\frac{V_0}{X_L} - \frac{V_0}{X_C}\right)^2} \Rightarrow$

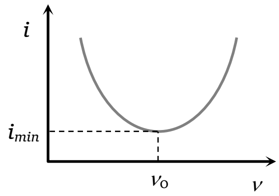
$$\frac{1}{Z} = Y = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2} = \sqrt{G^2 + (S_L - S_C)^2}$$

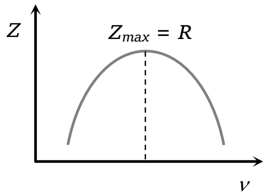
(3) **Resonance**

At resonance (i) $i_C = i_L \Rightarrow i_{min} = i_R$ (ii) $\frac{V}{X_C} = \frac{V}{X_L} \Rightarrow S_C = S_L \Rightarrow \Sigma S = 0$

(iii) $Z_{max} = \frac{V}{i_R} = R$ (iv) $\phi = 0 \Rightarrow$ p.f. = $\cos\phi$ = 1 = maximum (v) Resonant frequency $\Rightarrow \nu = \frac{1}{2\pi\sqrt{LC}}$

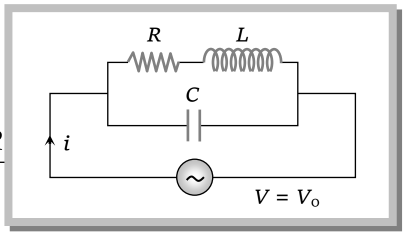
(4) **Current resonance curve**





(5) **Parallel *LC* circuits**

If inductor has resistance (R) and it is connected in parallel with capacitor as shown



(i) **At resonance**

(a) $Z_{\max} = \frac{1}{Y_{\min}} = \frac{L}{CR}$

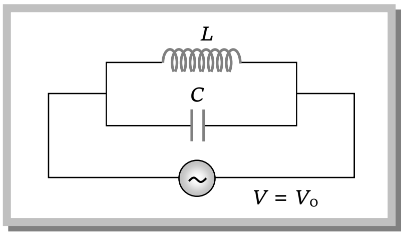
(b) Current through the circuit is minimum and $i_{\min} = \frac{V_0 CR}{L}$

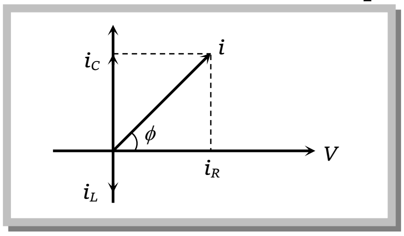
(c) $S_L = S_C \Rightarrow \frac{1}{X_L} = \frac{1}{X_C} \Rightarrow X = \infty$

(d) Resonant frequency $\omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}\ \frac{rad}{sec}$ or $\nu_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}\ Hz$ (Condition for parallel resonance is $R < \sqrt{\frac{L}{C}}$)

(e) Quality factor of the circuit $= \frac{1}{CR} \cdot \frac{1}{\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$. In the state of resonance the quality factor of the circuit is equivalent to the current amplification of the circuit.

(ii) **If inductance has no resistance :** If R = 0 then circuit becomes parallel *LC* circuit as shown





Condition of resonance : $i_C = i_L \Rightarrow \frac{V}{X_C} = \frac{V}{X_L} \Rightarrow X_C = X_L$. At resonance current i in the circuit is zero and impedance is infinite. Resonant frequency : $\nu_0 = \frac{1}{2\pi\sqrt{LC}}\ Hz$

Note :
- ❑ At resonant frequency due to the property of rejecting the current, parallel resonant circuit is also known as anti-resonant circuit or rejecter circuit.
- ❑ Due to large impedance, parallel resonant circuits are used in radio.

Concepts

- *Series RLC circuit also known as acceptor circuit (or tuned circuits or filter circuit) as at resonance it most readily accepts that current out of many currents whose frequency is equal to it's natural frequency.*
- *The choke coil can be used only in ac circuits not in dc circuits, because for dc frequency ν = 0 hence $X_L = 2\pi\nu L = 0$, only the resistance of the coil remains effective which too is almost zero.*
- *Choke coil is based on the principle of wattless current.*

Example

***Example*: 32** In a series circuit $R = 300\,\Omega, L = 0.9H$, $C = 2.0\mu F$ and $\omega = 1000\ rad/sec$. The impedance of the circuit is

[MP PMT 1995]

(a) 1300 Ω (b) 900 Ω (c) 500 Ω (d) 400 Ω

Solution: (c) $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{(300)^2 + \left(1000 \times 0.9 - \frac{1}{1000 \times 2 \times 10^{-6}}\right)^2} \Rightarrow Z = \sqrt{(300)^2 + (400)^2} = 500\,\Omega$.

***Example*: 33** In *LCR* circuit, the capacitance is changed from *C* to 4*C*. For the same resonant frequency, the inductance should be changed from *L* to

(a) 2*L* (b) *L*/2 (c) *L*/4 (d) 4*L*

Solution: (c) By using $\nu_0 = \frac{1}{2\pi\sqrt{LC}} \Rightarrow L \propto \frac{1}{C} \Rightarrow \frac{L'}{L} = \frac{C}{C'} = \frac{C}{4C} \Rightarrow L' = \frac{L}{4}$.

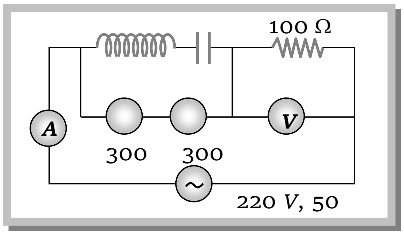
***Example*: 34** An *LCR* series circuit is connected to an external *e.m.f.* $e = 200\sin 100\pi t$. The values of the capacitance and resistance in the circuit are $2\mu F$ and $100\,\Omega$ respectively. The amplitude of the current in the circuit will be maximum when the inductance is

(a) 100 *Henry* (b) $50/\pi^2$ *Henry* (c) 100 π *Henry* (d) $100 \times \pi^2$ *Henry*

Solution: (b) Current will be maximum in resonance *i.e.* $X_L = X_C \Rightarrow 100\pi \times L = \frac{1}{100\pi \times 2 \times 10^{-6}} \Rightarrow$

$L = \frac{50}{\pi^2}$ *Henry*.

***Example*: 35** In the circuit shown below, what will be the readings of the voltmeter and ammeter



(a) 800 *V*, 2*A*

(b) 300 *V*, 2*A*

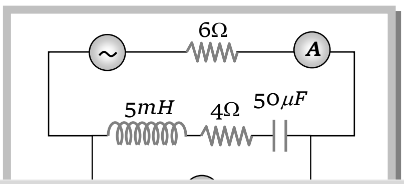
(c) 220 *V*, 2.2 *A*

(d) 100 *V*, 2*A*

Solution: (c) $V_L = V_C$; This is the condition of resonance and in resonance $V = V_R$ = 220 *V*.

In the condition of resonance current through the circuit $i = \frac{V_{rms}}{R} = \frac{220}{100} = 2.2A$.

***Example*: 36** In the circuit shown in the figure the ac source gives a voltage $V = 20\cos(2000\ t)$. Neglecting source resistance, the voltmeter and ammeter reading will be



(a) $0V, 0.47\,A$

(b) $1.68\,V, 0.47\,A$

(c) $0V, 1.4A$

(d) $5.6V, 1.4A$

Solution: (d) $X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\,\Omega$ and $X_C = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\,\Omega$

Total impedance of the circuit $= 6 + \sqrt{(R)^2 + (X_L - X_C)^2} = 6 + \sqrt{(4)^2 + 0} = 10\,\Omega$

Ammeter reads *r.m.s.* current so it's value $i_{rms} = \frac{V_{rms}}{\text{Total impedance}} = \frac{20/\sqrt{2}}{10} = \sqrt{2} = 1.41\,A$

Since $X_L = X_C$; this is the condition of resonance and in this condition $V = V_R = iR = 1.4 \times 4 = 5.6\ V$.

***Example*: 37** In a series resonant *LCR* circuit, if *L* is increased by 25% and *C* is decreased by 20%, then the resonant frequency will

(a) Increase by 10% (b) Decrease by 10% (c) Remain unchanged (d)Increase by 2.5 %

Solution: (c) $\nu_0 = \frac{1}{2\pi\sqrt{LC}}$ $\Rightarrow$ In this question $L' = L + 25\%$ of $L = L + \frac{L}{4} = \frac{5L}{4}$ and $C' = C - 20\%$ of C $= C - \frac{C}{5} = \frac{4C}{5}$

Hence $\nu_0' = \frac{1}{2\pi\sqrt{L'C'}} = \frac{1}{2\pi\sqrt{\frac{5L}{4} \times \frac{4C}{5}}} = \frac{1}{2\pi\sqrt{LC}} = \nu_0$

***Example*: 38** The self inductance of a choke coil is 10 *mH*. When it is connected with a 10*V* dc source, then the loss of power is 20 *watt*. When it is connected with 10 *volt* ac source loss of power is 10 *watt*. The frequency of ac source will be

(a) 50 *Hz* (b) 60 Hz (c) 80 *Hz* (d) 100 *Hz*

Solution: (c) With dc : $P = \frac{V^2}{R} \Rightarrow R = \frac{(10)^2}{20} = 5\,\Omega$; With ac : $P = \frac{V_{rms}^2 R}{Z^2} \Rightarrow Z^2 = \frac{(10)^2 \times 5}{10} = 50\ \Omega^2$

Also $Z^2 = R^2 + 4\pi^2\nu^2L^2 \Rightarrow 50 = (5)^2 + 4(3.14)^2\nu^2(10 \times 10^{-3})^2 \Rightarrow \nu = 80\ Hz$.

***Example*: 39** An ideal choke takes a current of 8*A* when connected to an ac source of 100 *volt* and 50*Hz*. A pure resistor under the same conditions takes a current of 10*A*. If two are connected in series to an ac supply of 100*V* and 40 *Hz*, then the current in the series combination of above resistor and inductor is

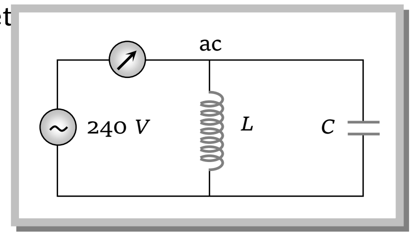
(a) 10*A* (b) 8*A* (c) $5\sqrt{2}$ *amp* (d) $10\sqrt{2}$ *amp*

Solution: (c) $X_L = \frac{V_{rms}}{i_{rms}} = \frac{100}{8} = 2\pi \times 50 \times L \Rightarrow L = \frac{1}{8\pi}$ *Henry* and $R = \frac{100}{10} = 10\,\Omega$

So impedance of the series *RC* circuit at a frequency of 40 *Hz* is $Z = \sqrt{\left(\frac{1}{8\pi} \times 2\pi \times 40\right)^2 + 10^2} = 10\sqrt{2}$

Hence current in the *RC* circuit now $i = \frac{E}{Z} = \frac{100}{10\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}A$.

***Example*: 40** In the following circuit diagram inductive reactance of inductor is 24Ω and capacitive reactance of capacitor is 48Ω, then reading of ammeter will be



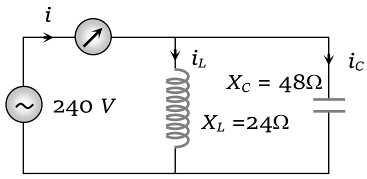
(a) 5 A

(b) 2.4 A

(c) 2.0 A

(d) 10 A

Solution: (a) $i_L = \frac{240}{24} = 10\ A$



$i_C = \frac{240}{48} = 5A$

Hence $i = i_L - i_C = 5A$

Tricky example: 4

In an LCR circuit $R = 100$ *ohm*. When capacitance C is removed, the current lags behind the voltage by $\pi/3$. When inductance L is removed, the current leads the voltage by $\pi/3$. The impedance of the circuit is

(a) 50 *ohm* (b) 100 *ohm* (c) 200 *ohm* (d) 400 *ohm*

Solution : (b) When C is removed circuit becomes RL circuit hence $\tan\frac{\pi}{3} = \frac{X_L}{R}$ .....(i)

When L is removed circuit becomes RC circuit hence $\tan\frac{\pi}{3} = \frac{X_C}{R}$ .....(ii)

From equation (i) and (ii) we obtain $X_L = X_C$. This is the condition of resonance and in resonance $Z = R = 100\Omega$.